# Prompting Mathematical Knowledge for Parenting

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# Abstract

This paper describes a mathematics-focused parental involvement program and its related impact on K-8 children and parents. Qualitative analysis consisting of parent, child, and teacher interviews and three-year quantitative testing showed that the treatment students improved significantly, whereas the comparison students did not. Moreover parents and teachers improved on measures of mathematical knowledge for teaching and attitude toward mathematics. We hypothesize that improved parent content and pedagogical knowledge, better attitudes towards mathematics, and improved parent-child interactions around mathematics motivate children to learn at school. Furthermore, we found aspects of parent knowledge and dispositions gained through the program to be analogous to teacher mathematical knowledge for teaching; we termed these aspects *mathematical knowledge for parenting*.

### Background

Parental involvement has been found to impact student achievement; however, parents are often not accessed as resources for helping children learn mathematics in standards-based school environments (Jackson & Remillard 2005; Perissini 1998). In this report, we describe a study of a parental involvement program designed to enrich schools mathematically.

With initial funding from the National Science Foundation (NSF) during the period from 1999-2003, the Math and Parent Partners (MAPPS) program was developed to engage K-8 parents in exploring with peers concepts, skills, and pedagogies behind the mathematics that their children are learning in the schools. The target population of MAPPS from the outset has been parents with students in economically disadvantaged schools, and qualitative evidence suggests improved student performance in mathematics (Henderson & Mapp 2002). Complementary outcomes included enhanced parent-parent, parent-child, parent-teacher, parentschool, teacher-student, and teacher-teacher relationships, notably among a high percentage of Latino participants. Further outcomes involved improved teacher and parent confidence with mathematics, parent enjoyment of mathematics, and parents encouraging high-level performance (See Bernier, Snider, & Civil 2003; Civil 2000, 2001, 2002; Civil, Andrade, & Anhalt 2000; Civil, Guevara, & Allexsaht-Snider 2002; Civil, Bernier, & Quintos 2003; MAPPS 2013; & Snider & Bernier 2003). This study (not conducted by the MAPPS Center) built on previous MAPPS research by asking the following research questions.

Does parental involvement in a standards-based mathematics program such as MAPPS carried on at economically disadvantaged K-8 schools improve student understanding and achievement in mathematics? Secondarily we ask, How might this improvement occur? In particular,

1. Do parents and teachers develop mathematical knowledge for teaching? If so, in what ways?

### 2. Do parents' and teachers' attitudes related to mathematics improve?

In this paper, we focus primarily on the *student* and *parent* improvements and developments.

# Literature Review, Rationale, and Theoretical Framework

In this section, we first review literature on parental involvement programs in mathematics. Second, we provide a rationale for this study. Third, we review literature that frames this study, that details our focus on mathematical knowledge for teaching, and that describes our theoretical framework.

# Parental Involvement Programs in Mathematics.

Aside from MAPPS, there have been several math-focused parental involvement programs enacted in locations such as the United States, Canada, Australia, and the United Kingdom (UK). Examples are the Mathematics Education Collaborative (Ferndale, WA), the Family Math Project (EQUALS, UC Berkeley), Family Math-ESSO (University of Western Ontario), and the Family Maths Project, Australia (FAMPA). FAMPA, based on the Family Math Project (US), offers short workshops for parents on school-level mathematics activities designed to build a mutual understanding of mathematics and its uses and to improve children's attitudes and problem solving skills around mathematics. Results from the FAMPA have shown a positive change in some participants' attitudes towards mathematics and the program (Horne 1998). From qualitative participant feedback, the Family Math-ESSO reported improved parent confidence, attitude, and understanding of mathematics (Onslow, Edmunds, Adams, Waters, & Chapple 2002; Edmunds 2004). In addition, a UK-based Family Numeracy program was implemented for parents and their 3- to 5-year-old children to increase the level of parental support in numeracy at home (Brooks & Hutchison 2002). Children attended both joint and separate sessions with their parents in the pilot that included a numeracy curriculum and bridging activities to expand numeracy in the home. The results revealed children's superior numeracy, support from parents, and attendance.

Moreover, as result of participation, some parents sought additional education. MAPPS differs from these programs in the depth and intensity of its engagement of parents in doing mathematics and its involvement of teachers as equal learners with parents in the program.

# Rationale

As research from the above programs amplifies, studies have shown that parent involvement in their children's education is strongly linked with children's academic outcomes (D'Agostino, Hedges, Wong, & Borman 2000; Epstein 1994; Kellaghan, Sloane, Alvarez, & Bloom 1993; Pomerantz, Moorman, & Litwack 2007). Henderson & Mapp (2002) stated, "The evidence is consistent, positive and convincing: families have a major influence on their children's achievement. When schools, families, and community groups work together to support learning, children tend to do better in school, stay in school longer, and like school more" (p. 7). Contrary to this evidence, some schools give increased parental involvement a token effort. Furthermore, attempts to induce parents' involvement and sustaining those efforts have not, to date, produced widespread documented positive effects (Horne 1998). Extensive reviews of evaluations comparing the effectiveness of programs with and without parent components (White, Taylor, & Moss 1992), and programs geared toward promoting stronger parent involvement (Mattingly, et al. 2002), have indicated that efforts to increase parents' involvement have had mixed results and not been consistently effective. This lack of consistently effective literature on parental involvement and especially parental involvement in mathematics gives rise to the rationale for this study. Namely, we investigated parental involvement and student achievement as well as factors that might stimulate that achievement. **Theoretical Framework** 

Literature framing our work arose from both theories of parental involvement as well as theories about teacher knowledge. Low-income parents may be untapped resources for the mathematical achievement of their children. Henderson, Mapp, Johnson, and Davies (2007) stated that districts serious about closing the achievement gap would have to address the school culture gap between schools that expect parents to remain at home versus schools that value families as resources. Like Henderson et al. (2007), Jackson and Remillard (2005) explained that schools often fail to recognize parents' involvement in their children's learning beyond the school walls. They called for schools to shift to a "parent-centric" view of parental involvement that valued parents' out-of-school contributions to their children's learning (p. 67). Moreover, they said, "Distinguishing between parent involvement in children's learning and in their schooling is critical to understanding parents' potential as intellectual resources for their children" (p. 69). Another concern related to parents of low socioeconomic status is that the reform-minded mathematics content and instruction advocated by the National Council of Teachers of Mathematics (NCTM) differs from ways in which many parents learned in school (Jackson & Epstein 2006). This change in instruction has caused conflict between parents and schools (Peressini 1998). Consequently, parents may feel less able and less welcome to assist their children with mathematical tasks, impeding student achievement (Remillard & Jackson 2006). The MAPPS program endorses the parent-centric view and seeks to improve student understanding and achievement in mathematics by providing parents with instruction that will help them assist their children with mathematics outside school walls (Bernier, et al. 2003).

# Mathematical knowledge for teaching and student achievement

The secondary research focus for this study was to ascertain *how* a math-focused parental involvement program might prompt student understanding and achievement. To this end, we asked *In what ways do parents develop mathematical knowledge for teaching*? The framework of

mathematical knowledge for teaching (MKT) relates to the knowledge and habits of mind needed to teach mathematics well (Ball, Thames, & Phelps 2008). In the framework, MKT includes six constructs of which we focused on the following four. Common content knowledge (CCK) is basic, lay-person knowledge of the mathematical content. Specialized content knowledge (SCK) is the way the mathematics arises in classrooms, such as for building representations. Knowledge of content and students (KCS) indicates a teacher's knowledge about how students think in mathematical contexts. Knowledge of content and teaching (KCT) indicates a teacher's knowledge of effective examples or teaching sequences. MKT encompasses both content knowledge (CCK & SCK) and pedagogical content knowledge (KCS & KCT).

The rationale for our focus on MKT of parents is two-fold. First, MKT has been linked to student achievement, and second, teachers in low socio-economic status schools tend to possess lower MKT than their economically advantaged counterparts (Hill, Rowan, & Ball 2005). Although the literature on MKT is about classroom teachers, we hypothesize that MKT in some form may be developed with parents and that this "parental" MKT may impact student understanding and achievement. Our choice to employ MKT to analyze data in this study was driven by parents and teachers acting as co-learners and co-partners for the betterment of children's understanding and achievement of mathematics within the MAPPS environment.

# **Emergent Perspective**

A theoretical framework which includes both a sociocultural aspect and a constructivist aspect is needed to underpin a study on the development of parent, teacher, and child knowledge and attitudes in a collaborative environment such as MAPPS. Cobb and Yackel (2004) describe the *emergent perspective* as a version of social constructivism which coordinates interactionism and psychological constructivism. Indeed, "... social norms and beliefs are seen to be reflexively

related such that neither exists independently of the other" (p. 212). The emergent perspective served as a framework for this study because the development of MKT is related to the development of classroom social norms as outlined in the emergent perspective (Ball 2003; Cobb & Yackel 2004). The MAPPS program fostered a collaborative learning environment where social norms could develop alongside and be constrained by the reorganization of beliefs.

# Methods

# **Context for the Study**

The core MAPPS activity on which data for this study were collected and analyzed was the 8-week *Math for Parents Mini-courses*. Each Mini-course is based on a theme of school mathematics and each involves a parent for eight 2-hour sessions over the course of two to three months. The Mini-course curriculum was developed in the NSF MAPPS grant and focuses on the following five content domains (one domain per Mini-course): *Thinking About Numbers* (offered 2x), *Thinking About Fractions, Decimals and Percents* (offered 3x), *Geometry for Parents, Thinking About Patterns* (offered 1x), and *Data for Parents* (offered 1x) (MAPPS 2013).

Mini-courses engage parents and teachers in doing mathematics using hands-on materials, working in small groups to solve problems, and presenting their solutions to the whole gathering as outlined by the NCTM process standards (NCTM 2000). For example, participants were instructed to form a collection of color tiles that was 10% blue, 15% green, 50% red, and 25% yellow (Knapp, Jefferson, & Landers 2013). Of importance to the program, the parents are not taught rote procedures or advanced coursework. Both content knowledge and pedagogical content knowledge are intertwined into the instruction for parents, with pedagogical

considerations made relevant by Mini-course instructors to the particular grade levels of children of participating parents and teachers (Ball, Thames, & Phelps 2008).

# **Participants**

The study was conducted on a high-poverty, suburban MAPPS program in the Southeast during 2008-2011. The county in which the study was conducted had a population of 58,000 and was 65.8% White/Anglo, 32.1% African American, and 2.5% Latino. Sixty-five percent of students in the district were eligible for free and reduced lunch. Total enrollment in the four participating elementary schools was 2270. Approximately 75% of MAPPS attendees were single parents, and those who attended the Mini-courses did so with one to three children. Most of the parents had graduated from high school with some technical training, and they typically held low-income jobs. Attendees were approximately 40% Caucasian, 40% African-American, and 20% Hispanic.

Schools were selected for participation based on their Title I status, willingness of the principals to participate, and superintendent advisement. All parents, teachers, paraprofessionals, and children from the selected schools were invited to participate, regardless of their past participation in MAPPS. While their parents were in class, children aged preschool-Grade 3 were invited to play educational games from the MAPPS curriculum or from the *Investigations in Number, Data, and Space* curriculum (Akers et al. 1998). Children in 4<sup>th</sup> -8<sup>th</sup> grade attended the Mini-courses alongside their parents. Over the course of three years, eight separate 8-week Mini-courses were offered. Mini-courses were hosted by the local university's Office of Continuing Education; instructors were graduate students in mathematics education who were practicing teachers. Teachers attending MAPPS were offered a stipend and Professional Learning Units.

Data analyses focused on a group of treatment parents, teachers, and children who attended at least half of an 8-week Mini-course. In all, 115 children, 59 parents, and 33 teachers attended at least one Mini-course on a regular basis. In addition, nearly twice that many individuals attended sporadically. A matched comparison group of children, from the same four schools, was investigated as well (n = 89). Comparison students were chosen from among participating schools' after-school programs or by availability for testing during the school day.

# Methodology

For the first research question, "Does parental involvement in MAPPS improve student understanding and achievement," we chose to employ a quasi-experimental design because parents and teachers in this study self-selected to the program. (Shadish, Cook, & Campbell 2002; Vogt 2007). For the secondary research question, "How might this improvement occur," we employed a multi-tiered teacher development experiment (TDE) ( Lesh & Kelly 2000; Presmeg & Barrett 2003). The TDE takes a global approach to studying teachers' development as well as observing and analyzing the learning of the teachers' students. We extend the methodology to study teachers' and parents' development and the learning of their children. The researchers coordinated the analysis of the levels of the experiment by engaging in an iterative model of reflection and interaction following each year of the study.

# **Data Analysis**

Data collected for the study included pre/post surveys, pre/post-tests of mathematical knowledge for teaching (MKT) (Hill, Schilling, & Ball 2004), and pre/post attitude surveys (Tapia 1996) given to parents and teachers before and after each Mini-course. Ninety-five interviews of willing parents, teachers, and children were collected as well; most interviews lasted approximately 15 minutes and were audiotaped and transcribed. Forty-five of the interviews were of parents, 34 were of teachers, and 16 were of children. Five of the teachers

interviewed were MAPPS instructors. A project-designed, free response test of 7-17 questions, depending on the children's age and relating directly to MAPPS Mini-course content, was given to children before and after five of the Mini-courses. Children's productions created during the Mini-courses were analyzed as well. Finally, children's mathematics test scores from the state Criterion Reference Competency Test (CRCT) were collected for years 2008-2011.

Our analysis occurred on three levels: the Mini-course level, the yearly level, and the cumulative level. Attendance records were kept so that we could compare effects of sustained involvement in the program with short involvement. For quantitative analysis of CRCT scores, individual students were the unit of analysis because we were looking at the improvement of students' learning on their own and their parents' training in MAPPS. CRCT scores in mathematics were compared using paired samples *t*-tests to assess whether the treatment students improved significantly over the prior years compared to the comparison students. In addition, performance levels (exceeds: 3, meets: 2, does not meet: 1) were compared using paired samples *t*-tests unless data were not normally distributed, in which case the Sign Test for Matched Pairs (Hays 1994) was used. Free response test items and children's artifacts were analyzed for student understanding by the first author (See Figure 4).

For the research focus on parental and teacher development of MKT, we utilized domainspecific multiple-choice Content Knowledge for Teaching Mathematics (CKT-M) measures as pre and post-tests corresponding to the content for each Mini-course (Hill, Schilling, & Ball 2004). Reliability on the CKT-M measures was previously established for in-service elementary and middle school teachers (Hill 2007); that reliability of the measures has not been established with parents is a limitation of this study. We conducted paired samples *t*-tests on the scaled scores from the CKT-M tests generated through Item Response Theory (IRT) to assess for improved domain-specific knowledge.

The 95 interviews and pre/post surveys were analyzed for development of MKT, evidence of student understanding and achievement, changes in attitudes towards mathematics, and other factors seemingly related to student understanding and achievement. Interview questions were such as these: 1) Have you learned anything about mathematics that you did not know before? Explain. 2) Have you learned anything in MAPPS that helped you help your child or students with math? Explain. 3) Has participation in MAPPS impacted your feelings about math or your child's math instruction at school? The MKT framework (CCK, SCK, KCT, KCS) was used to analyze data. In other words, we coded for instances of developing MKT in addition to other factors such as those social norms in the emergent perspective resembling MKT and factors arising from the parental involvement literature that might prompt student achievement. Examples of social norms that resemble the components of MKT include, "explaining and justifying solutions, attempting to make sense of explanations given by others, indicating agreement and disagreement, and questioning alternatives in situations in which a conflict in interpretations or solutions had become apparent" (Cobb & Yackel 2004 p. 212). Several of our codes included CCK, SCK, KCT, KCS, parent-child interaction, learning community, and enjoyment (See Table 1). Open coding was thus employed to ascertain how the MKT domains were developing. The first and second authors as well as a graduate student in mathematics education coded the qualitative data so that each interview was coded by at least two people. Coders then compared coding results and resolved discrepancies in coding for each interview. Each year, the list of open codes was revised, resulting in 59 codes that were clustered and compressed. Primary and secondary codes were separately identified for parents, teachers, and children. At the end of each year and at the conclusion of the study conducted cross-case analysis (Coffey & Atkinson 1996).

For attitude analysis, the researchers administered a modified version of the Attitudes Toward Mathematics Inventory (Tapia & Marsh 2004) to parents and teachers along with the CKT-M test before and after each Mini-course. The inventory consisted of 25 items to reflect five affective mathematics dimensions (confidence, anxiety, value, enjoyment, and motivation). Parents were asked to rate statements such as "Mathematics is a very interesting subject" on a Likert scale (Tapia 1996). Attitude surveys were analyzed using paired samples *t*-tests. The ATMI was found to be reliable (alpha = .948) for parents and teachers.

# **Results and Discussion**

After coding the interviews and pre/post surveys, we tallied the 59 codes to identify the salient areas of participant growth as well as factors prompting that growth. Results from pre/post surveys confirmed interview results. The primary and secondary results are shown in Tables 1-4. To identify primary and secondary results, we looked for clusters in the data each year. The cluster of codes with high frequencies became primary for the particular year, and the secondary cluster was likewise identified. To be regarded as *primary* in the final analysis, a code had to have been in either the yearly primary or secondary categories *all three years*. To be regarded as secondary in the final analysis, a code had to have been in a yearly primary or secondary list at least once. This process corrected for varying numbers of interviews each year. Lastly, codes were compressed. For aggregate results in Table 4, results across participants were tallied before primary and secondary decisions were made.

Table 1

Results from 16 Children Interviews

Code (all Primary)	Freq
Improved Parent-Child Interaction	16
Enjoyment of MAPPS	15
Student learning/achievement	8

Table 2

Code	Freq	Description of Result
Primary:		
Improved Parent-Child Interaction	86	Interactions improved
Content Knowledge	56	Primarily CCK for parents
CCK(26), SCK(10), GLM* (20)		*GLM-General learning of mathematics expressed,
		but could not be identified as CCK or SCK
Enjoyment of MAPPS	46	
Valuing MAPPS	40	High value placed on program
Knowledge of Content and Tchg	30	КСТ
Secondary:		
Continuing Education	23	Desire to continue education expressed
Student learning/achievement	22	
Confidence	16	Confidence with mathematics and helping children

Results from 45 Parent Interviews

# Table 3

Results from 34 Teacher Interviews

Code	Freq	Description of Result
Primary:		
Knowledge of Content and Teaching (KCT)	56	
Content Knowledge	28	Primarily SCK for teachers
SCK(16) CCK(6) *GLM(6)		*GLM-General learning of mathematics
Valuing MAPPS	28	High value placed on program
Learning Community	23	The learning community was important.
Secondary:		
Enjoyment of MAPPS	12	
Broader impact of program	10	Program impacted non-MAPPS students
Student learning/achievement	8	

Table 4

Aggregate Results from 95 Interviews

Code	Freq
Primary:	
Improved Parent-Child Interaction	103
Knowledge of Content and Teaching	87
Content Knowledge CCK(32) SCK(29) GLM (26)	87
Enjoyment of/Valuing MAPPS	75
Valuing MAPPS	75
Learning Community	43
Student learning/achievement	42
Secondary:	

Confidence/Motivation	31
Continuing Education	23
Broader Impact of MAPPS	18

To innumerate the results in Tables 1-4, we first present baseline data on the participants, detailing their levels of knowledge, attitudes toward mathematics, and their purposes for attending MAPPS sessions garnered from interview and survey data. We then provide quantitative results and qualitative narrative relating to each research question. We conclude with a discussion of the results. Examples of our coding process are embedded within the qualitative results in bold font.

# Baseline

Consistent with the findings of Jackson and Remillard (2005), many parents attending MAPPS assisted or desired to assist their children with learning mathematics on some level. Parents often attempted to assist with homework, and others purchased skill-based practice workbooks or flashcards for their children to complete. Parents reported considerable consternation with the homework process. For example, one mother reported incorrectly helping in the following interview.

Int:OK. So, do you help him with his math homework sometimes?Parent A:Yes. But lately he doesn't want me to help him. Remember a couple of weeks<br/>ago I was telling you about the tenths and ten?Int:Yes

Parent A: And I did it for him, but I was doing the tenths instead of ten. And we got all of them wrong.

**Math phobia**, or strong dislike and avoidance of mathematics, was reported repeatedly by parents and also served as an impediment to helping with mathematics homework. Yet throughout the interview process, parents displayed a strong undercurrent of **valuing mathematics**, **awareness** of the usefulness of mathematics, and **desire** to help their children become successful in mathematics. The tension between a compulsion to help but lacking the ability to help was evidenced by the following interview with a father who said, "You know it's going to be a day when she comes home, and I really ain't going to know what to say or do." The parent further shared that due to his poor mathematics grades; he had not been allowed to play sports in school. He said, "It came very kind of depressing because you know all I ever dreamed about was playing basketball and baseball. It was very discouraging. And I ended up quitting [school]." Parents such as this father may have mathematics phobia and low content knowledge, but they desire better for their children, so they attended MAPPS.

Another motivator for parents to attend MAPPS was as a mode of self-improvement to strengthen their own content knowledge. We found that parents had a **self-awareness** of their own knowledge, strategies, and limitations. One mother stated, "You want to help your children and help yourself...why not?" She continued, "In five years, you might be in a university for real." Finally, some parents attended MAPPS as a way of supporting their school and improving their children's CRCT scores. Later in the program, purposes for attending were enjoyment of the program, pressure from a child who was enjoying the program, **valuing the program**, and free food. Teachers attended because the program was something exciting happening at the university, a stipend was offered, and the principals influenced them to attend.

# **Research Question: Did Student Understanding and Achievement of Mathematics**

# Improve?

Evidence of improved student understanding and achievement presented in the form of parent, teacher, and child self-report during interviews, free response tests, and CRCT scores. When asked, children mentioned specific concepts that they were learning in MAPPS. Parents and teachers also reported improvements. For example, a parent reported helping her child figure out practice problems for the CRCT using the fraction chart from MAPPS. Also, when asked, "Do you think that as a result of you attending the MAPPS classes that this has helped improve your students' performance in math," one teacher replied as follows:

Teacher: Absolutely. We studied the data a lot this year from one benchmark to the next and then comparing last year's CRCT scores to this year's CRCT. And overall, my students' scores in mathematics improved. I guess if you do a mean score across the board, overall, they improved by 35 points each as a mean score for the class.

Children likewise demonstrated improved understanding of mathematics through their

free response test such as on the pre/post item displayed in Figures 4 and 5 (Griffin 2007).

Figure 4. Pre-test response.

Shade 60 % of this grid.



Figure 5. Post-test response.

Shade 60 % of this grid.



The student gained conceptual **understanding** that each fifth of the grid represented 20% of the whole.

# **CRCT** results

The primary quantitative result from the MAPPS study was that MAPPS students taking at least one Mini-course over the three years improved significantly on the mathematics portion of the CRCT. A paired samples *t*-test was used (n = 39, p < 0.001, d = 0.766). Comparison students *did not* improve significantly (n = 36, p = 0.331) (See Table 5 and Figure 5.).

Table 5Three Year Mean CRCT Changes 2008-2011

Three Tear Mean CKer Changes 2000 2011									
	2008	2011	Difference in CRCT scores						
Comparison $(n = 36)$	817.4	823.1	+5.7						
	sd 31.7	sd 29.9	sd 33.2						
Treatment $(n = 39)$	807.4	825.5	+18.1						
	sd 22.5	sd 25.34	sd 19.2						

*Note.* The 95% confidence interval for mean improvement in CRCT scores for the treatment group was [-24.35, -11.9]. Both 2008 and 2011 data sets were checked for normality using the Anderson-Darling test (p = 0.855, p = 0.128 respectively).

# Yearly comparisons of CRCT averages.

We additionally present one-, two-, and three-year comparisons of CRCT scores to display score differences over time. Analysis of treatment student scores showed greater, yet not significant gains over the comparison students in one-year comparisons. Two-year increases were *significant* for the treatment groups and *mixed* for the comparison groups (See Tables 8 and 9). By year three, the treatment group improved significantly across 2008-2011, whereas, the comparison group did not (See Tables 10 and 11). Moreover, the *improvement* of the treatment group was significantly higher than the comparison improvement in 2008-2011 (p = 0.055).

Table 8 Two-Year Comparisons: Treatment Group

<u>1wo-rec</u>	ar Coi	nparisons:	<u>i reatment</u>	Group					
	п	Pre	Post	Sig?	Pre	Post	Sig?	Mean	Mean
		CRCT	CRCT		PL	PL		CRCT	PL
		Mean	Mean		Mean	Mean		increase	increase
2008-	31	810.1	819.2	Yes	1.7	2.0	Yes	+9.1	+0.3
2010				<i>p</i> =			<i>p</i> = 0.007		
				0.086			sign test		

			<i>d</i> = 0.306			<i>d</i> = 0.551		
2009- 47 <sup>a</sup> 2011	813.5	826.2	Yes p = 0.001 d = 0.455	1.8	2.0	Yes p = 0.057 Sign test d = .282	+12.7	+0.2

<sup>a</sup> Participants took at least one Mini-course in 2008-2011. Note that the number of participants included in the statistical analysis is not the same as the number of attendees due to the availability of test scores. For example, K-2 students were not tested all 3 years using the CRCT due to budget cuts. Score availability was also limited by transient students and the willingness/availability of participants to take the assessments. Furthermore, some comparison students began attending MAPPS due to ongoing recruitment efforts and thus were no longer considered part of the comparison group. Finally, participants self-selected which years they attended. Because of these fluctuations, the *n* value varied from year to year.

		- T	ison Grou	P				
п	Pre	Post	Sig?	Pre	Post	Sig?	Mean	Mean
	CRCT	CRCT		PL	PL		CRCT	PL
	mean	mean		mean	mean		increase	increase
41	817.3	825	No	1.9	2.0	No	+7.7	+0.1
46	821.7	829.9	Yes	1.9	2.1	Yes $p =$	+8.2	+0.2
			<i>p</i> =			0.035		
			0.023			Sign test		
			d =			d =		
			0.249			0.296		
	41	CRCT mean 41 817.3	CRCT meanCRCT mean41817.3825	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccc} CRCT & CRCT & PL \\ mean & mean & mean \\ \hline 41 & 817.3 & 825 & No & 1.9 \\ \hline 46 & 821.7 & 829.9 & Yes & 1.9 \\ & & & & & \\ \end{array}$	CRCTCRCTPLPLmeanmeanmeanmean41817.3825No1.92.046821.7829.9Yes1.92.1 $p =$ 0.023d =0.023	CRCTCRCTPLPLPLmeanmeanmeanmean41817.3825No1.92.0No46821.7829.9Yes1.92.1Yes $p =$ $p =$ 0.0350.023Sign test $d =$ $d =$ $d =$	CRCTCRCTPLPLPLCRCTmeanmeanmeanmeanmeanincrease41817.3825No1.92.0No+7.746821.7829.9Yes1.92.1Yes $p =$ +8.2 $p =$ 0.0350.023Sign test $d =$ d =

Table 9Two-Year Comparisons: Comparison Group

CRCT scores across two-year spans increased by amounts of 9.1 and 12.7 for treatment students and by 7.7 and 8.2 for comparison students (See Tables 8 & 9). Performance level increases were larger for the treatment group as well (0.3 and 0.2 vs 0.1 and 0.2). Values of 1, 2, and 3 were assigned to the corresponding levels and then averaged.

In the 3-year comparison, treatment students' CRCT scores increased significantly by an average of 18.1 points; comparison students increased by an average of 5.7 points, which was not a significant increase (See Tables 10 & 11). Performance levels (PL) for the treatment group also improved

significantly, improving on average by 0.3 levels versus a non-significant mean improvement of 0.1 levels for the comparison group. In looking at the Performance level means, it appears that MAPPS mainly helped the Level 1 (does not meet) students move to Level 2 (meets). Performance level comparisons were conducted using the Sign Test for the Median rather than paired samples t tests because normality of the data could not be established.

	п	Pre	Post	Sig?	Pre	Post	Sig?	Mean	Mean
		CRCT	CRCT		PL	PL		CRCT	PL
		Mean	Mean		Mean	Mean		increase	increase
2008-	39 <sup>a</sup>	807.4	825.5	Yes	1.7	2.0	Yes $p =$	+18.1	+0.3
2011				<i>p</i> <			0.004		
				0.001			Sign test		
				d =			d =		
				0.766			0.557		

Table 10Three-Year Comparison: Treatment Group

<sup>*a*</sup> Participants took at least one Mini-course in 2008-2011.

## Table 11

Three Vear	Comparison:	Comparison	Group
Intee-teur	Comparison.	Comparison	Oroup

	п	Pre CRCT	Post CRCT	Sig?	Pre PL	Post PL	Sig?	Mean CRCT	Mean PL
		mean	Mean		mean	mean		increase	increase
2008- 2011	36	817.4	823.1	No	1.9	2.0	No	+5.7	+0.1

CRCT score increases of students who took two or more Mini-courses were at times higher than those who took only one. However, students did not consistently perform better when they took two or more Mini-courses. Further research is needed to determine the effects of multiple Mini-courses on student achievement.

In sum, qualitative data reported by all three groups of participants-parents, teachers, and children-revealed student understanding and **achievement** as a top code. Analysis of quantitative CRCT data triangulated this finding, showing significant improvement over the

course of three years. Our next research question investigated how these improvements might have come about.

# **Research Question #2: How Might This Improvement Occur? In Particular, Do Parents** and Teachers in MAPPS Develop Mathematical Knowledge for Teaching?

In this section, we discuss the development of "parental" MKT based on qualitative interviews and quantitative tests. We focus our analysis on three aspects of MKT based on their emergence in the data, namely, Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), and Knowledge of Content and Teaching (KCT).

# **Common Content Knowledge**

Both parents and teachers developed Content Knowledge; however, parents'

development occurred more in the area of CCK than SCK (See Tables 2 and 3). During the interviews, parents gave numerous examples of new content (CCK) that they had learned due to MAPPS, such as turning percents into fractions, calculating the volume of a cylinder, and that a nonzero number to the zero power is one. Parent A, mentioned in the baseline section, said the following:

Parent A:	For example, one night we had this conversation: A halfwhat is the half of a
	quarter?
Int:	oh.
Parent A:	and would you believe that for years I didn't know that half of a quarter
Int:	half of a quarter
Parent A:	It is one eighth.
Int:	yes.
Parent A:	and that you keep cutting it [the fraction strip]ummm1/2 of 1/8
Int:	so…you know. Ok
Parent A:	and even on this test [technical college entrance exam] that I got, they asked me that question $1/2$ of a question and L could answer
	me that question, $1/2$ of a quarter, and I could answer

Parent A learned conceptually that <sup>1</sup>/<sub>2</sub> of <sup>1</sup>/<sub>4</sub> is 1/8 while engaging in a fraction strip activity, and she subsequently was able to answer a related question on her college entrance exam. Parents shared that their increased content knowledge prepared them to assist their children with specific homework tasks and also strengthened the parents' confidence to assist with homework. Moreover, the parents' increased **CCK** and **confidence** transferred into a desire to continue their own education (**Continuing Education**).

Throughout the three years of the study, parents attended MAPPS to benefit their children. Yet as they participated, participants shared a desire and new confidence to continue their education, whether it is to get their GED, a degree, or some other certification. Toward the end of the program, we saw numerous participants *act* on that desire and begin college. MAPPS appeared to empower participants and provided a bridge to higher education for both parents and teachers. We mention this result here because student achievement has been linked to higher education levels of parents (Choy 2001), thus addressing the research question, *How might student improvement occur*?

The qualitative result that parents improved their content knowledge was substantiated by the CKT-M test results. The Number and Operations Mini-course was offered during Year 1 and Year 2. The Fractions, Decimals & Percents Mini-courses was offered each year. The same Number and Operations test (or alternate form) was given before and after each of these Minicourses. Most individual 8-week Mini-courses produced increased means. Significant changes of the parent and teacher group was noted when the first to last Number and Operations Minicourse scores were compared (See Table 14).

	п	Pre IRT	Post IRT	Change	Sig?
				in st dev	
1 <sup>st</sup> -Last Mini-course Parents & Teachers	60	-1.21361	-0.96921	0.24440	YES
					p = 0.029
					d = 0.282
1 <sup>st</sup> -Last Mini-course Parents only	40	-1.35844	-1.18154	0.17690	NO

Table 14

Number & Operations Content Knowledge Tests (CKT-M)

*Note.* The 95% confidence interval for mean improvement in CKT-M IRT scores was [-0.462, - 0.027] P&T. Both pre and post data sets were checked for normality using the Anderson-Darling test (p = 0.504, 0.311 respectively-P&T; p = 0.507, 0.530 respectively T).

The content knowledge tests were designed such that a well-prepared elementary teacher would get 50% of the questions correct, which would be an Item Response Theory (IRT) scaled score or standard deviation of 0. The average scores for both parents and teachers increased.

# **Knowledge of Content and Teaching**

The third aspect of mathematical knowledge for teaching that developed for parents during MAPPS was Knowledge of Content and Teaching (KCT) (See Tables 2-4). For parents, we found that the ability to help children with mathematics homework involved more than mere content knowledge. KCT for parents involved improving their teaching efforts toward their children in both formal homework tasks as well as informal day-to-day mentioning of mathematics (Jackson & Remillard 2005).

In MAPPS classes, parents' own strategies were valued, prompting parents to value their children's mathematical strategies. Likewise, parent and teacher explanations were shared with the entire group, modeling for them the importance of eliciting children's reasoning. Another area of development for parents was in the use of manipulatives. Parents' and teachers' entry level on manipulative use differed, but the MAPPS instructors adapted their approach to the needs of participants by introducing and modeling tasks and sequences of instruction with appropriate manipulatives. One parent gave evidence of improved Knowledge of Content and Teaching (**KCT**) in that she learned to explain addition using base ten blocks:

Int: So what specifically did she [child] learn better with you just using the base ten blocks with her?

Parent C: The order...let's say in the tens place where she had something like 10 plus 10. Um, a lot of times, she would struggle because I would try to use pennies or little dots on a paper, and she didn't understand it. She would get confused, and I would get upset. And it wasn't going anywhere, but when we got the blocks or the little units or whatever, she was able to understand... This parent learned that the pre-grouped manipulative, base-ten blocks, better assisted her daughter with place value concepts in multi-digit addition than ungrouped pennies or drawings (Van de Walle, Karp, & Bay-Williams 2010). Parents in the program tended to learn about manipulatives for the first time; whereas teachers often knew of them, but not how to use them well. To sum up parents' development, we found eight "parental" aspects of mathematical knowledge for teaching (See Figure 7).

- 1. Content knowledge (CCK)+
- 2. Valuing students' own strategies
- 3. Listening to students' explanations
- 4. Knowing that there is more than one way to solve a problem
- 5. Knowing to use manipulatives versus solely pencil and paper to solve problems
- 6. Knowing *how* to use manipulatives to model problems (SCK)
- 7. Knowing appropriate games and skill reinforcers
- 8. Knowing how to support the learning process (i.e., Do not immediately give the answer. Work within a child's Zone of Proximal Development (Vygotsky 1978))

Figure 7. Parental aspects of mathematical knowledge for teaching.

Improvements in parents' Knowledge of Content and Teaching in these areas gave rise to

improved parent-child interaction around mathematics, relating to how the MKT improvement

might have occurred.

# **Research Question #2: How Might This Improvement Occur?**

# **Parent-child interaction**

The next theme that presented from interview data involved **parent-child interactions** 

around mathematics, and it confirms and extends prior qualitative research that MAPPS

strengthens family relationships (Bernier et al. 2003). Whereas many parents previously had

expected their children to work on mathematics homework in isolation, they began assisting their

children with homework and further engaging the children in mathematical thought at home.

The following child interview provides evidence of this result.

# Int:Are they [your parents] better at explaining now that they've come to MAPPS?Child:YesInt:How?Child:They tell about the shapes and the stuff that I do at school. They compare it to here.

Instead of shying away from helping their children, parents began enjoying the challenge and felt confident enough in their mathematical skills to figure out mathematics problems and tasks. Parent D said, "I feel much more confident working with Sarah (pseudonym), because even though it's new and it's a different way of presenting the material, this class is helping me to learn how to help her." Parents' focus began to shift from their children *completing* mathematics homework to *understanding* mathematics homework.

For some, this time of homework interaction evolved into "family time." Parents and children shared MAPPS games and activities at home to reinforce the concepts and skills learned in the Mini-courses. Parents were provided cut-out manipulatives such as base-ten blocks, pattern blocks, and tangrams for this purpose. Parent D additionally said,

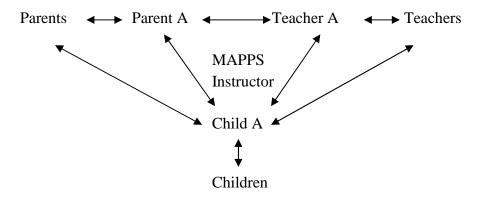
...those tangrams? I LOVE those. Those were our [her and her daughter] favorites. Trying to put the pictures. Figure out how they go. We had the best time with those. We played with those all the time, even though it wasn't homework, you know?"

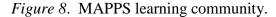
Another parent explained that MAPPS helped her *listen* to her child and thereby improve homework time. She said, "It showed me to listen at her as to how she's trying to tell me, and then I can see whether or not she's getting to the right answer or not, or going about it the right way." For this parent, instead of trying to explain a concept using the parent's own strategy, which may have been different from how the child was thinking about it and independent of how it was taught at school, the parent listened to the child's strategy. Thus, as one parent explained, MAPPS assisted parents in giving children "one-on-one" assistance. Another aspect of improved parent-child interaction occurred on-site during the MAPPS Mini-courses. Numerous parents expressed that the MAPPS environment provided enjoyable "family time." One teacher observed about children, "...they really enjoyed getting it [the problem] before their family member did and impress them with their knowledge and all the other parents. They enjoyed getting up and showing how they found the answer." At times, parents were surprised to see their "shy" children boldly sharing knowledge with the group that the parent did not know the child had. Families engaged in playful competition in seeing who could get the problems correct. One mother exemplified the family aspect of MAPPS by saying, "We all as a family are graduating tonight."

# Learning community

In addition to strengthening parent-child relationships, the learning community afforded by MAPPS strengthened parent-teacher relationships. Bonding formed because parents got to know teachers in a different way than in a negatively-connoted position of power, telling parents what to do or not to do in regard to their children. Teachers and parents enjoyed a level playing field in which all were learning for the desired end of helping children. The parents' and teachers' interaction with the 4<sup>th</sup>-8<sup>th</sup> grade children during Mini-courses provided them an on-site clinical experience with teaching using their newfound knowledge. Parents appreciated teachers' extra effort to help children learn, and teachers came to view parents as dedicated individuals, invested in the academic success of their children. The light-hearted nature of the Mini-courses drew families and teachers back for not only more mathematics learning, but relationships fueled by a desire to learn mathematics.

Benefits of this learning community environment for parents and children were threefold. First parents, teachers, and children assisted one another in learning mathematics. At times, *children helped parents* figure out problems, which became a source of pride and motivation for the children, especially when they could present their solutions to the group. Hence, the MAPPS environment forged a Parent-Teacher-Child triangle of knowledge and respect (See Figure 8). The arrows in the figure represent interactions within the MAPPS learning community.





The MAPPS instructor is in the foreground, impacting and facilitating the learning community. Parent A, Teacher A, and Child A interact with children, teachers, and other parents. Knowledge was impacted, constructed, and shaped by interactions among participants.

Enjoyment of mathematics was the second product of the learning community. "Before you leave, you're laughing because you've learned. The average 8 and 78-year-old learning together," said one parent. She explained that people come to MAPPS for the **enjoyment** of learning. Children as well expressed enjoyment such as in seeing their teacher and parent interact.

The third product of the learning community included **motivation**. The learning community motivated parents to do better for their children, in terms of interaction around mathematics, because they saw other parents doing it. As one parent put it, she was motivated to explicitly budget time at home for helping her child with mathematics. Other motivational factors for children included 1) mere parental presence at MAPPS, 2) parental interest in what

the children were doing, 3) a non-traditional, ungraded learning environment at the university,

and 4) a location that they found exciting. Seeing their parents value mathematics also

motivated children to value it as well.

The MAPPS-fostered relationship between teacher and parent motivated children not just

to learn mathematics at MAPPS, but in school as well. Teachers reported that when MAPPS

children encountered MAPPS content in their daytime classrooms, they were more motivated

and confident than previously. One teacher reported,

I think as they [children] saw things that we did outside of class [at MAPPS] in our class, it motivated them because they could share their experience with their peers. They were able to say, 'I understand this because I've seen it before.' It built their confidence. It definitely built their confidence because when they knew how to do something, people [other MAPPS participants] looked to them for help.

A child's interview sums up the improved **motivation** and **confidence**:

Int:Would you encourage other people to join these classes if you had a chance to?Child:YesInt:You would. OK, like friends at school: What would you tell them?Child:I would say there's a university class, and you would get smarter and feel more<br/>confident. You would just have fun with that.

Thus, parental involvement in mathematics coupled with the learning community of the

MAPPS environment appeared to spawn increased motivation and confidence for children to learn in the school setting. This may explain the delay in improved CRCT scores for students. Analysis of the qualitative data appears to show that the MAPPS environment improved classroom learning for children, an improvement which would impact student achievement over time. The quantitative data bears this out, indicating significant improvements by the third year of the study.

# Research Question #3: Do Parents' Attitudes Related to Mathematics Improve?

In light of the qualitative evidence about improved confidence and motivation of parents with respect to mathematics learning (their own) and teaching (of their children), the quantitative attitude survey serves as triangulation. As with the content knowledge tests, mean **attitude** scores improved during most sessions.

Parents and teachers as a group improved significantly on the attitude toward mathematics survey when comparing the first time they took the survey to the last (some participants took several Mini-courses and thus took the survey multiple times) (p = 0.084, d = 0.129). An increase in parent attitude toward mathematics may have contributed to the improved motivation of children to learn mathematics (See Table 16).

Table 16. Parent Versus Teacher Attitude Scores- 125 points possible.

	п	Pre	Post	Change	Significance?
1 <sup>st</sup> -Last Mini-course Parents & Teachers	65	93.1	95.2	2.2	YES
		sd 17.7	sd 16.0	sd 9.9	p = 0.084 d = 0.129

*Note.* The 95% confidence interval for the mean improvement in attitude scores was [-4.60, 0.30] P&T. Both pre and post data sets were checked for normality using the Anderson-Darling test ( pre p = 0.664, post p = 0.309).

# Conclusions

In conclusion, we revisit our research questions in light of the literature and the emergent perspective. First, children constructed mathematical knowledge as they interacted with other children during MAPPS sessions and with their parents at home after the sessions. Parents reported children's grades improving as well as better understanding of children's mathematics homework. Teachers likewise reported improved mathematics understanding among MAPPS students. Third, children displayed increased understanding of MAPPS content through the project-designed, free response tests and CRCT scores. This result substantiates and builds on prior qualitative MAPPS evidence and the UK-based Family Numeracy Program that parental involvement in mathematics boots student performance (Brooks & Hutchison 2002; Henderson & Mapp 2002).

Several factors seemed to indirectly impact student understanding and achievement. Analysis of the data in this study revealed that parents have a desire to help their children with mathematics, and that they value mathematics learning for their children. However, similar to the findings of Remillard and Jackson (2006), parents do not have the language or tools, and sometimes, the mathematics knowledge and confidence to support their children's learning. The collaborative MAPPS environment stimulated social construction of parents' Content Knowledge and Knowledge of Content and Teaching, aspects of MKT. Content knowledge strengthened parents' confidence and attitudes towards mathematics. Knowledge of Content and Teaching strengthened their ability to explain their knowledge to children, especially through improved choice of and appropriate use of manipulatives. Couched in interactionism, the improved Content Knowledge and KCT appeared to strengthen children's understanding and achievement of mathematics.

Moreover, we found through MAPPS that certain aspects of mathematical knowledge for teaching (MKT) seemed germane to parents' mathematical work with their children in the home setting. Of course, homework help and informal mathematics instruction such as games are done in the context of the home environment. But we contend that the crux of the improved mathematics help at home was in part due to relationships fostered by the mathematics-focused parental involvement program. *Children's interactions with parents fueled by the MAPPS learning community prompted children's motivation and confidence to learn mathematics at school, leading to student achievement gains.* Children's construction of mathematical knowledge was facilitated and constrained by social interaction with their parents as key players.

Also impacting the parent-child relationship and thereby impacting student achievement were parents' improved attitudes toward mathematics and confidence in explaining it. This result substantiates the attitude results from the Family Math Project (Horne 1998). Parentteacher relationships forged through the learning community also impacted student motivation and consequently sustained mathematics learning. Thus, although several aspects of MKT for parents had a counterpart to MKT for teachers, the critical, math-focused, relationship between parents and children seemed to demand a separate construct. Consequently, we advocate that elements of MKT relating to parents be described as *Mathematical Knowledge for Parenting*, as opposed to "parental" Mathematical Knowledge for Teaching. In using *Mathematical Knowledge for Parenting*, we inherently assert that there is important mathematical work that need occur between parents/guardians and children and cannot be replaced by work between teachers and children. Our study implies that aspects of Mathematical Knowledge for Parenting can and should be taught in parental involvement programs such as MAPPS and that such programs should directly involve children. It is incumbent upon schools to *partner with parents* in the mathematics education of their children.

Finally, we found through this study that parents and teachers attended MAPPS with dual purposes. They wanted to help children, and they wanted to help *themselves*. As much as participants enjoyed the learning community, they took the learning environment seriously as a means of self-improvement. And this study verifies that MAPPS did in fact advance parents' and teachers' knowledge and teaching ability, and notably, confidence to continue their own education, a result similar to the findings of the UK-based Family Numeracy program (Brooks & Hutchison 2002). MAPPS provided a way to break the generational cycle of math phobia and incompetence as well as opening lines of communication that enhanced the mathematics learning culture of schools.

Thus, although parents, teachers, and children came to MAPPS with widely varying background knowledge, the learning community afforded all participants opportunities to learn and develop mathematically, and for the parents and teachers, to learn and develop pedagogically. Hence, we believe that this study implies that when parental involvement programs are paired with professional development for teachers, children and schools benefit. Further study is needed on the constitution and impact of parents' *Mathematical Knowledge for Parenting*.

# Limitations

It is a limitation to this study that parents self-selected to the program. In addition, additional time spent on mathematics, whatever the intervention, could have impacted the results. However, the recruitment effort that MAPPS and school personnel engaged in to encourage parents and teachers to attend the program was extensive. Recruitment included information sessions, repeated phone calls, knocking on doors, and constant reminders sent home with children. We believe that these recruitment efforts impacted parents' choices to attend. Furthermore, due to continued advertising, some students in the control group joined the program. Thus, we believe that the results were due at least in part to our math-focused parental involvement intervention.

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