MATHEMATICAL KNOWLEDGE FOR PARENTING

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Parents, K-8 teachers, and 4th-8th grade children participated as equals in math-focused parental involvement through the Math and Parent Partners (MAPPS) program. Pre/post testing and qualitative interviews revealed that MAPPS served as a platform for improvement in content knowledge of participating parents. We hypothesize that improved parent content and pedagogical knowledge, better attitudes towards mathematics, and improved parent-child interactions around mathematics motivate children to learn at school. Furthermore, we found aspects of parent knowledge and dispositions gained through the program to be analogous to teacher mathematical knowledge for teaching; we termed these aspects "mathematical knowledge for parenting".

Keywords: Mathematical Knowledge for Teaching, Informal Education, Attitudes

Background and Research Questions

Student achievement lags in many economically disadvantaged schools. Two factors associated with this achievement gap include inadequate teacher knowledge and low parental involvement (Hill, Rowan, & Ball, 2005; Jackson & Remillard, 2005). A school district in the Southeast partnered with the local university to boost student achievement in Title I schools through the Math and Parent Partners (MAPPS) program. We asked,

> Does parental involvement in a standards-based mathematics program such as MAPPS carried on at Title I K-8 schools improve student understanding and achievement in mathematics? Secondarily we asked, how might this improvement occur? In particular, Do parents and teachers in MAPPS develop mathematical knowledge for teaching?

Students were found to improve standardized test scores significantly over a three year period (author). However, this paper focuses on factors that may have prompted the student improvement. In particular, we describe *parents'* development in mathematical knowledge for teaching as they participated in MAPPS learning communities.

Theoretical Framework and Literature Review

Hill, Rowan, and Ball (2005) reported a study in which teachers' *mathematical knowledge for teaching* (MKT) was linked to student achievement in first and third grade. Moreover, they found that teachers in economically disadvantaged schools tended to possess lower MKT. The framework of mathematical knowledge for teaching (MKT) relates to the knowledge and habits of mind needed to teach mathematics well (Ball, Thames, and Phelps, 2008). In the framework, MKT includes six constructs of which we focused on the following four in investigating the Math and Parent Partners learning communities. Common content knowledge (CCK) is basic, lay-person knowledge of the mathematical content. Specialized content knowledge (SCK) is the way the mathematics arises in classrooms, such as for building representations. Knowledge of content and students (KCS) indicates a teacher's knowledge about how students think in mathematical contexts. Knowledge of content and teaching (KCT) indicates a teacher's

knowledge of advantageous representations or teaching sequences. MKT encompasses both content knowledge (CCK & SCK) and pedagogical content knowledge (KCS & KCT).

Studies have additionally shown that parent involvement in their children's education is linked with children's academic outcomes (D'Agostino, Hedges, Wong, & Borman 2000; Epstein 1994; Kellaghan, Sloane, Alvarez, & Bloom 1993). As Henderson and Mapp (2002) stated, "The evidence is consistent, positive and convincing: families have a major influence on their children's achievement. When schools, families, and community groups work together to support learning, children tend to do better in school, stay in school longer, and like school more" (p. 7). Low-income parents may be untapped resources for the mathematical achievement of their children. Henderson, Mapp, Johnson, and Davies (2007) asserted that districts serious about closing the achievement gap would have to address the school culture gap that expects parents to remain relatively uninvolved in their children's mathematics learning. Although parental involvement may be linked to student achievement, parents are often not accessed as resources for helping children learn mathematics in standards-based school environments (Jackson & Remillard, 2005; Perissini, 1998). In this paper, we describe a study of a parental involvement program that engaged parents, children, and teachers in mathematics learning.

Participants and Context

The Math and Parent Partners (MAPPS) program equips families to act as mathematical resources for their children and for schools. MAPPS curriculum was developed with National Science Foundation funding to engage K-8 parents in exploring with peers the concepts and skills behind the mathematics that their children are learning in schools (See http://mapps.math.arizona.edu/). Currently, the MAPPS program serves sites in six states and the Virgin Islands. One MAPPS site, located in the Southeast and the focus of this article, worked toward improving the mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008) of both parents and teachers in Title I schools within its school district. All parents, teachers, paraprofessionals, and children from selected schools were invited to participate. The local university partnered with MAPPS and the school district to offer Mini-courses for parents and teachers, while young children participated in related mathematical activities and games. Children in 4^{th}-8^{th} grade accompanied their parents in the Mini-course classes. Mini-course sessions convened two hours per week for eight weeks. Over the course of three years, eight separate 8-week Mini-courses, centered on the National Council of Teachers of Mathematics' (NCTM) (2000) content and process standards, were offered. These Mini-courses were hosted by the University's Office of Continuing Education, and instructors were graduate students in mathematics education who were also practicing teachers.

8-week Mini-course Title	NCTM Content Standard Addressed
Thinking About Numbers (offered two times)	Number & Operations
Thinking About Fractions, Decimals, and Percents (offered 3x)	Number & Operations
Thinking in Patterns (offered once)	Algebra
Geometry for Parents (offered once)	Geometry and Measurement
Data for Parents (offered once)	Data Analysis & Probability

Figure 1: Math for Parents Mini-course Curriculum

In all, 115 children, 59 parents, and 33 teachers from primarily four Title I elementary schools attended at least one Mini-course on a regular basis. Nearly twice that many participants

attended sporadically. Approximately 75% of attendees were single parents, and those that attended the Mini-courses did so with one to three children. Most of the parents had graduated from high school with some technical training, and they typically held low-income jobs. Attendees were approximately 40% Caucasian, 40% African-American, and 20% Hispanic. Tea

MAPPS Mini-courses engage parents in doing mathematics using hands-on materials, working in small groups to solve problems, and presenting their solutions to the whole group as outlined by the NCTM process standards (NCTM, 2000). For example, participants were instructed to form a collection of color tiles that was 10% blue, 15% green, 50% red, and 25% yellow (author). Both content knowledge and pedagogical content knowledge are intertwined into the instruction for parents (Ball, Thames, & Phelps, 2008), with pedagogical considerations made relevant by Mini-course instructors depending on grade levels of participating children.

Data Analysis

To assess the impact of the MAPPS Mini-courses, parents and teachers took pre/post tests on mathematical knowledge for teaching (Hill, Schilling, & Ball, 2004) and pre/post attitude surveys (Tapia, 1996). Pre/post tests and surveys were administered before and after each 8-week Mini-course. A focus group of parents, teachers, and children also participated in 95 pre/post interviews. Interviews lasted approximately 15 minutes, and questions were such as these: 1) Have you learned anything about mathematics that you did not know before? Explain. 2) Have you learned anything in MAPPS that helped you help your child or students with math? Explain. Interviews were coded for evidence of improved student understanding, achievement, and factors that might affect that improvement, such as the elements of mathematical knowledge for teaching: CCK, SCK, KCT, SCK (See Table 1). After coding the interviews and pre/post surveys, we tallied the 59 codes to identify the salient areas of participant growth as well as factors prompting that growth. We looked for clusters in the data each year, producing primary and secondary results for each year. At the end of the study, we compressed codes and identified themes based on the primary and secondary codes. Themes arising from the coding process included improved parent-child interaction around mathematics, stronger content knowledge, enjoyment of and valuing the MAPPS program, and development of KCT.

Table 1: Results from 45 Parent Interviews

Code	Freq	Description of Result
Primary:		
Improved Parent-Child Interaction	86	Interactions improved
Content Knowledge	56	Primarily CCK for parents
CCK(26), SCK(10), GLM* (20)		*GLM-General learning of mathematics expressed, but could not be identified as CCK or SCK
Enjoyment of MAPPS	46	
Valuing MAPPS	40	High value placed on program
Knowledge of Content and Tchg	30	KCT
Secondary:		
Continuing Education	23	Desire to continue education expressed
Student learning/achievement	22	
Confidence	16	Confidence with mathematics and helping children

Results and Discussion

We present interview data from several parents to amplify our coding process and themes that emerged. Examples of codes are in bold.

Content Knowledge

During the interviews, parents gave numerous examples of new content (CCK) that they had learned due to MAPPS, such as turning percents into fractions, calculating the volume of a cylinder, and that a nonzero number to the zero power is one. A Parent said the following:

Int: OK. So, do you help him with his math homework sometimes?

Parent: Yes. But lately he doesn't want me to help him. Remember a couple of weeks ago I was telling you about the tenths and ten?

Int: Yes

Parent: And I did it for him, but I was doing the tenths instead of ten. And we got all of them wrong.

Int: Oh, no!

Parent: What ever time I try to help him, he says to me, "Mommy, I don't think you know what you're saying." So I have a problem, right there. But now that I'm coming here, I can show him my notes and say "This is what they taught me."

Int: So did you sit down and talk about that?

Parent: Yes, we did. The last time I came here I did, um, addition, and then he took it to school and it was right. So, he's kind of trusting me a little now. [Both laugh] So that's good. I'm trying not to mess up again.

This parent also shared a second example of content that she had learned.

Parent: For example, one night we had this conversation: A half…what is the half of a quarter?

Int: oh.

Parent: and would you believe that for years I didn't know that half of a quarter…

Int: half of a quarter

Parent: It is one eighth.

Int: yes.

Parent: and that you keep cutting it [the fraction strip]…ummm…1/2 of 1/8…

Int: so…you know. Ok

Parent: and even on this test [technical college entrance exam] that I got, they asked me that question, 1/2 of a quarter, and I could answer…

The parent learned conceptually that ½ of ¼ is 1/8 while engaging in a fraction strip activity, and she subsequently was able to answer a related question on her college entrance exam. Parents shared that their increased content knowledge prepared them to assist their children with specific homework tasks and also strengthened the parents' confidence to assist with homework. Moreover, the parents' increased **CCK** and **confidence** transferred into a desire to continue their own education (**Continuing Education**).

The qualitative result that parents improved their content knowledge was substantiated by the CKT-M test results. The Number and Operations Mini-course was offered during Year 1 and Year 2. The Fractions, Decimals & Percents Mini-courses was offered each year. The same Number and Operations test (or alternate form) was given before and after each of these Mini-courses. Most individual 8-week Mini-courses produced increased means. Significant changes

of the parent and teacher group was noted when the first to last Number and Operations Mini-course scores were compared (See Table 2).

Table 2: Number & Operations Content Knowledge Tests (CKT-M)

	n	Pre IRT	Post IRT	Change in st dev	Sig?
1st-Last Mini-course Parents & Teachers	60	-1.21361	-0.96921	0.24440	YES $p = 0.029$ $d = 0.282$
1st-Last Mini-course Parents only	40	-1.35844	-1.18154	0.17690	NO

Note. The 95% confidence interval for mean improvement in CKT-M IRT scores was [-0.462, -0.027] P&T. Both pre and post data sets were checked for normality using the Anderson-Darling test ($p = 0.504$, 0.311 respectively-P&T).

The CKT-M tests were designed such that a well-prepared elementary teacher would get 50% of the questions correct, which would be an Item Response Theory (IRT) scaled score or standard deviation of 0. The average scores for both parents and teachers increased.

Knowledge of Content and Teaching

The third aspect of mathematical knowledge for teaching that developed for parents during MAPPS was Knowledge of Content and Teaching (KCT) (See Table 1). For parents, we found that the ability to help children with mathematics homework involved more than mere content knowledge. KCT for parents involved improving their teaching efforts toward their children in both formal homework tasks as well as informal day-to-day mentioning of mathematics (Jackson & Remillard 2005).

In MAPPS classes, parents' own strategies were valued, prompting parents to value their children's mathematical strategies. Likewise, parent and teacher explanations were shared with the entire group, modeling for them the importance of eliciting children's reasoning. Another area of development for parents was in the use of manipulatives. Parents' and teachers' entry level on manipulative use differed, but the MAPPS instructors adapted their approach to the needs of participants by introducing and modeling tasks and sequences of instruction with appropriate manipulatives. One parent gave evidence of improved Knowledge of Content and Teaching (**KCT**) in that she learned to explain addition using base ten blocks:

> *Int*: So what specifically did she [child] learn better with you just using the base ten blocks with her?
>
> *Parent:* The order…let's say in the tens place where she had something like 10 plus 10. Um, a lot of times, she would struggle because I would try to use pennies or little dots on a paper, and she didn't understand it. She would get confused, and I would get upset. And it wasn't going anywhere, but when we got the blocks or the little units or whatever, she was able to understand…

This parent learned that the pre-grouped manipulative, base-ten blocks, better assisted her daughter with place value concepts in multi-digit addition than ungrouped pennies or drawings (Van de Walle, Karp, & Bay-Williams 2010). Parents in the program tended to learn about manipulatives for the first time; whereas teachers often knew of them, but not how to use them well. To sum up parents' development, we found eight "parental" aspects of mathematical knowledge for teaching (See Figure 2).

1. Content knowledge (CCK)
2. Valuing students' own strategies
3. Listening to students' explanations
4. Knowing that there is more than one way to solve a problem
5. Knowing to use manipulatives versus solely pencil and paper to solve problems
6. Knowing *how* to use manipulatives to model problems (SCK)
7. Knowing appropriate games and skill reinforcers
8. Knowing how to support the learning process (i.e., Do not immediately give the answer. Work within a child's Zone of Proximal Development (Vygotsky 1978))

Figure 2: Parental Aspects of Mathematical Knowledge for Teaching

Improvements in parents' Knowledge of Content and Teaching in these areas gave rise to improved parent-child interaction around mathematics, relating to how the MKT improvement might have occurred.

Parent-Child Interaction

The next theme that presented from interview data involved **parent-child interactions** around mathematics. Whereas many parents previously had expected their children to work on mathematics homework in isolation, they began assisting their children with homework and further engaging the children in mathematical thought at home. The following child interview provides evidence of this result.

Int: Are they [your parents] better at explaining now that they've come to MAPPS?
Child: Yes
Int: How?
Child: They tell about the shapes and the stuff that I do at school. They compare it to here.

Instead of shying away from helping their children, parents began enjoying the challenge and felt confident enough in their mathematical skills to figure out mathematics problems and tasks. A parent said, "I feel much more confident working with Sarah (pseudonym), because even though it's new and it's a different way of presenting the material, this class is helping me to learn how to help her." Parents' focus began to shift from their children *completing* mathematics homework to *understanding* mathematics homework.

For some, this time of homework interaction evolved into "family time." Parents and children shared MAPPS games and activities at home to reinforce the concepts and skills learned in the Mini-courses. Parents were provided cut-out manipulatives such as base-ten blocks, pattern blocks, and tangrams for this purpose. Parent D additionally said,

> …those tangrams? I LOVE those. Those were our [her and her daughter] favorites. Trying to put the pictures. Figure out how they go. We had the best time with those. We played with those all the time, even though it wasn't homework, you know?

Another parent explained that MAPPS helped her *listen* to her child and thereby improve homework time. She said, "It showed me to listen at her as to how she's trying to tell me, and then I can see whether or not she's getting to the right answer or not, or going about it the right way." For this parent, instead of trying to explain a concept using the parent's own strategy, which may have been different from how the child was thinking about it and independent of how

it was taught at school, the parent listened to the child's strategy. Thus, as one parent explained, MAPPS assisted parents in giving children "one-on-one" assistance.

Another aspect of improved parent-child interaction occurred on-site during the MAPPS Mini-courses. Numerous parents expressed that the MAPPS environment provided enjoyable "family time." One teacher observed about children, "…they really enjoyed getting it [the problem] before their family member did and impress them with their knowledge and all the other parents. They enjoyed getting up and showing how they found the answer." At times, parents were surprised to see their "shy" children boldly sharing knowledge with the group that the parent did not know the child had. Families engaged in playful competition in seeing who could get the problems correct. One mother exemplified the family aspect of MAPPS by saying, "We all as a family are graduating tonight."

Enjoyment of mathematics and **motivation** were further qualitative results. "Before you leave, you're laughing because you've learned. The average 8 and 78-year-old learning together," said one parent. She explained that people come to MAPPS for the enjoyment of learning. Children as well expressed enjoyment such as in seeing their teacher and parent interact. The learning community further motivated parents to do better for their children, in terms of interaction around mathematics, because they saw other parents doing it. As one parent put it, she was motivated to explicitly budget time at home for helping her child with mathematics.

In light of the qualitative evidence about improved confidence and motivation of parents with respect to mathematics learning (their own) and teaching (of their children), the quantitative attitude survey serves as triangulation. As with the content knowledge tests, mean **attitude** scores improved during most sessions. Parents and teachers as a group improved significantly on the attitude toward mathematics survey when comparing the first time they took the survey to the last (some participants took several Mini-courses and thus took the survey multiple times) ($p = 0.084$, $d = 0.129$). An increase in parent attitude toward mathematics may have contributed to the improved motivation of children to learn mathematics (See Table 3).

Table 3: Parent Versus Teacher Attitude Scores- 125 Points Possible.

	n	Pre	Post	Change	Significance?
1st-Last Mini-course Parents & Teachers	65	93.1 sd 17.7	95.2 sd 16.0	2.2 sd 9.9	YES $p = 0.084$ $d = 0.129$

Note. The 95% confidence interval for the mean improvement in attitude scores was [-4.60, 0.30] P&T. Both pre and post data sets were checked for normality using the Anderson-Darling test (pre $p = 0.664$, post $p = 0.309$).

Conclusions

We found through MAPPS that certain aspects of mathematical knowledge for teaching (MKT) seemed germane to parents' mathematical work with their children in the home setting. Of course, homework help and informal mathematics instruction such as games are done in the context of the home environment. But we contend that the crux of the improved mathematics help at home was in part due to relationships fostered by the mathematics-focused parental involvement program.

Also impacting the parent-child relationship and thereby impacting student achievement were parents' improved attitudes toward mathematics and confidence in explaining it. Parent-teacher relationships forged through the learning community also impacted student motivation and consequently sustained mathematics learning. Thus, although several aspects of MKT for parents had a counterpart to MKT for teachers, the critical, math-focused, relationship between parents and children seemed to demand a separate construct. Consequently, we advocate that elements of MKT relating to parents be described as *Mathematical Knowledge for Parenting*, as opposed to "parental" Mathematical Knowledge for Teaching. In using *Mathematical Knowledge for Parenting*, we inherently assert that there is important mathematical work that need occur between parents/guardians and children and cannot be replaced by work between teachers and children. Our study implies that aspects of Mathematical Knowledge for Parenting can and should be taught in parental involvement programs such as MAPPS and that such programs should directly involve children. It is incumbent upon schools to *partner with parents* in the mathematics education of their children.

References

Ball, D. L., M. H. Thames, and G. Phelps. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education,* 59, no. 5, 389-407.

D'Agostino, J. V., L. V. Hedges, K. K. Wong, and G. D. Borman. (2000). Title I home-school partnerships: Effects on parenting practices and student achievement. In, *Title I: Compensatory Education at the Crossroads*, edited by G.D. Borman, S.C. Stringfield, and R.E. Slavin. Mahwah, NJ: Lawrence Erlbaum Associates.

Epstein, J.L. (1994). Theory to practice: School and family partnerships lead to school improvement and student success. In *School, Family, and Community Interaction: A View from the Firing Lines,* edited by C. Fagnano and B. Werber, pp. 39-52. Boulder, CO: Westview Press.

Henderson, A.T., and K. L. Mapp.(2002) *A New Wave of Evidence: The Impact of School, Family and Community Connections on Student Achievement.* Austin, TX: National Center for Family and Community connections with Schools, Southwest Educational Development Laboratory.

Henderson, A.T., K. L. Mapp, V. R. Johnson, and D. Davies. (2007). *Beyond the Bake Sale: The Essential Guide to Family-School Partnerships.* New York: The New Press.

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Education Research Journal, 42 (2),* 371-406.

Hill, H. C., Schilling, S. G. and Ball, D. L. (2004). Developing measures of teachers' mathematics knowledge for teaching. *The Elementary School Journal,105*, 11-30.

Jackson, K., and J. Remillard. (2005). Rethinking parent involvement: African American mothers construct their roles in the mathematics education of their children." *School Community Journal, 15(1),* 51-73.

Kellaghan, T., Sloane, K., Alvarez, B., & Bloom, B.S. (1993). *The home environment and school learning: Promoting parental involvement in the education of children*. San Francisco, CA: Jossey-Bass Publishers.

The National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Peressini, D.D. (1998). The portrayal of parents in the school mathematics reform literature: Locating the context for parental involvement. *Journal for Research in Mathematics Education, 29*(5), 555-582.

Tapia, M. (1996). *Attitudes Toward Mathematics Inventory.*